Revisiting capitalist reproduction: equilibrium, networks, and intersectoral competition

John Cajas-Guijarro[a]

[a] Central University of Ecuador and Latin American Faculty of Social Sciences FLACSO-Ecuador.

Email addresses: jcajasg@uce.edu.ec

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Abstract

This article revisits capitalist reproduction with a focus on equilibrium, networks, and competition. Thus, after a literature review, a three-stage scheme of capitalist reproduction is presented: *1)* Equilibrium in goods markets is shown to limit the sectoral distribution of employment; *2)* Monetary networks are proposed to reinterpret simple and extended reproduction using Markov chains, and *3)* The role of intersectoral competition is incorporated to explain the emergence of the long-run average rate of profit, and some factors that could hinder this emergence are brought to light. The article then presents the main conclusions and possible avenues for future research on the complexity of capitalist reproduction.

Keywords: sectoral schemes; equilibrium; monetary networks; intersectoral competition.

1.INTRODUCTION

One of Karl Marx's greatest *analytical* contributions to economics are his capitalist reproduction schemes– developed in volume two of *The Capital*- which describe the interactions between two productive sectors (separated into means of production and consumption), necessary to reproduce capital at a simple scale (scale and steady-state growth) and expanded (expansion driven by capitalist accumulation). The results are usually interpreted within the context of crisis[1] due to sectoral imbalance. For capital to reproduce without creating an imbalance in the markets, production sectors must maintain "appropriate proportions" which are difficult or even impossible to achieve, given the pressures of competition and other contradictions inherent in capitalism (Sweezy, 1942, pp. 156-162)

Capitalist reproduction schemes are relevant to economic thinking. Marx was inspired by Quesnay's economic table, in which he describes the necessary interactions for wealth produced by agriculture to be distributed among the different social classes (productive/agricultural, landlords, unproductive). Later, sectoral observations by Quesnay, Marx, von Bortkiewicz and others, alongside thinking on balance in general, inspired Leontief's input-output analysis (Baumol and ten Raa, 2009). Even Sraffa's (1960) subsistence and surplus production scheme could be considered akin to the Marxist theories of simple and expanded production.

Given the importance of the Marxist schemes and in an attempt to delve more deeply into their implications, this study revisits the reproduction of capital from the standpoint of balance, commodity chains and intersectoral competition. Section two reviews literature that highlights the relevance of reproduction schemes. Section three contains two subsections: the first presents starting theories and schemes for studying sectoral balance in capitalist reproduction, as well as presenting monetary circulation chains and Markov chains that reinterpret simple and extended reproduction. The second subsection combines Markovian interpretations with intersectoral competition to create an understanding of how the average rate of long-term profit emerged. Additionally, some factors that could hinder this emergence are shown, in accordance with concrete conditions of real capitalist competition. Finally, conclusions and suggestions for future discussion points are shared in section four.

2. LITERATURE REVIEW

Marx's schemes for capitalist reproduction prompted new thinking about the Marxist economy[2] , and a wealth of literature that – without wishing to exclude potential classifications – can be put into four groups: early commentary, analytic reinterpretations, synthesis attempts, and extensions. As Sweezy notes (1942, pp.162-166), Tugan-Baranovsky (1905) stands out amongst early commentators. Using a three-sector scheme he concludes that crisis occurs if part of capital's aggregated surplus value is not divided correctly. However, this crisis could be avoided as everything produced tends to be sold. Hilferding (2019) proposes a two-sector scheme where fluctuations and crisis occur as multiple lag variables and are included. Luxemburg (2015) criticized various aspects of the schemes, including the lack of clarity around money, which allows for sectoral expansion in extended reproduction (a potential solution being expansion towards non-capitalist territories) (Desai and Veneziani, 2009). In contrast with Tugan Baranovsky, Luxemburg states that an inherent problem of aggregated demand is always present in capitalism (Harcourt and Kriesler, 2016, p.256; Kalecki, 1967).

After these preliminary comments[3] - including Sweezy (1942), Robinson (1951), Kalecki (1968) and others- profound mathematical reinterpretations will be presented. According to Diaz and Velasco (2016) Harris (1972)[4] should be first in this second group; he describes how achieving balance simultaneously in production markets and consumption requires that labor exploitation, mechanization and sectoral distribution of jobs follow rules of proportionality that are difficult to adhere to, given that each aspect responds to different economic phenomena. Moreover, Harris concludes that exploitation is key to “closing” Marx’s sectoral scheme and building a consistent theory on profit (1972, pp. 518-519). Subsequently, the schemes underwent major mathematical reinterpretations. Some examples are, matrix generalizations by Roemer (1978), linking price formation to accumulation; the role of money and effective demand in extended reproduction, Foley (1983); thoughts on the movement of capital caused by differentials between the rates of return, Nikaidos (1983 and 1985); sectoral growth and the contradictions within capitalist reproduction (the growing gap between production and consumption, a growing economic difference between workers and capitalists, unemployment or job scarcity, and other cumulative imbalances), Okishio (1993)[5] , and other additional literature.

Likewise, many works of classical-Marxist economic synthesis have discussed reproduction schemes. This third group of writings contains works such as the dual system, Morishima (1973 chs.9-12); numerical and algebraical representations, Koshimura (1975 ch VI); an explanation of crisis due to profit compression, underconsumption, and fiscal crisis, Roemer (1981, ch 9); dynamic models, Foley (1986, ch 5); the schemes’ possible Keynesian and Kaleckian elements, suggested by Trigg (2006, ch. 2-3); or recently combining the schemes with an input-output analysis and a graphic representation of monetary flow by Tsoulfidis and Tsaliki (2019, ch 2), and an economic-Marxist synthesis by Basu (2021 ch 3).

There is a more heterogenous body of writing collecting works that have extended Marxist schemes to discuss phenomena beyond balance and sectoral growth. Notably, works that have linked the schemes to cyclical dynamics such as Sherman (1971). He proposes various sectoral models that represent the contradictions caused by overproduction, concluding that widescale cyclical unemployment causes slower economic growth. Other cyclical interpretations were also proposed by Laibman (1992) who, using a variant of Goodwin’s model (1967) linked to Marx’s schemes, suggested a schematic representation of the periodical crisis that occur in capitalism, using simulation models.[6] This fourth collection of literature goes beyond crisis and includes a variety of extensions to Marxist schemes, such as their connection to the environment (Burkett, 2004), non-productive sectors (Olsen, 2015), probability interpretations of prices (Cockshott, 2016) and financial sectors (Veronese Passarella, 2019). Some even propose evolving sectoral approaches that integrate short-term Keynesian dynamics, (for example the use of installed capacity) with long-term classical-Marxist dynamics (for example, the average rate of earning) (Rotta, 2020).

This article seeks to contribute to the literature on the subject, by adding a reinterpretation of capitalist reproduction from a perspective that combines sectoral balance, monetary flow chains and intersectoral competition, as proposed by the following model.

3. MODEL

Balance and a Markovian interpretation of capitalist reproduction

Starting outline and sectoral balance

Harris (1972) provides some initial hypotheses. (A) Closed economy, with no government. (B) Two productive sectors exist, the sector $i = 1$ that produces means of production and the sector $i = 2$ that produces means of consumption (the subindex $i = 1, 2$ will always represent those two sectors). (C) Workers spend their entire salary purely on consumption, while capitalists can consume, save or both. (D) Salaries are paid at the end of production.[7] (E) All sectors produce during the same period. (F) There is always enough money for merchandise to circulate without liquidity issues, as it is assumed that there are two financial sectors (Fi, i = 1, 2) where the Fi sector only gives credit to the productive sector i (for example the productive sector i and the financial sector Fi belong to the same economic-financial group).[8] (G) All debts are paid in the same production period. (H) There is balance in all the markets if full use is made of the installed capacity. (I) There is no technical change. (J) Sector 1 is more modernized than sector 2. (K) Labor exploitation conditions are exogenous (defined during the “class struggle”) and equal in both sectors. (L) All workers earn the same nominal salary (free labor mobility exists). (M) The sectoral distribution of employment always allows for markets to balance. (N) All monetary quantities are measured in direct prices (Shaikh, 1977), with prices directly proportional to the work time that is socially necessary to produce merchandise. Thus, all quantities measured in money are governed by the same monetary expression of time worked (see Moseley, 2016). (O) Capital flow between sectors is not permitted. The hypotheses (N) and (O) describe the absence of intersectoral competition, that is to say, the absence of competition between capitalists from sectors that produce different goods (see Shaikh, 2016).

It is also advisable to define the constant capital-variable capital ratio k_i, the quotient profit-salaries ratio ϵ_i, and the rate of profit π_i measured only in relation to constant capital due to assumption (D), and the sectoral composition of variable capital is x:

$$k_i = \frac{C_i}{V_i} \qquad (1)$$

$$\epsilon_i = \frac{P_i}{V_i} \qquad (2)$$

$$\pi_i = \frac{P_i}{C_i} \qquad (3)$$

$$x = \frac{V_1}{V_2} \qquad (4)$$

Where C_iV_i represent -in money- constant capital (the value of means of production) and variable capital (the value of workforce used) respectively, and P_i represents profit, for sector *i*.

Given the assumption (N) of direct prices, it is true that:

$$C_i = \rho TTRSN_{C_i} \tag{5}$$

$$V_i = \rho TTRSN_{V_i} = wL_i \tag{6}$$

$$P_i = \rho TTRSN_{P_i} \tag{7}$$

Where ρ is the monetary expression of time worked, $TTRSTN_{Ci}$ and $TTRSTN_{Vi}$ respectively represent the socially necessary working time to produce the means of production represented by C_i and the means of consumption represented by V_i. *w* is the nominal hourly salary per hour worked, L_i is the number of workforce hours used and $TTRSTN_{Pi}$ is the surplus labor or socially necessary working time used to produce goods that cover capitalist profit in direct prices (Marx, 2009a)

There are some observations regarding k_i. For example, rewriting (1) using (5) and (6) the following is obtained:

$$k_i = \frac{C_i}{V_i} = \frac{\rho TTRSN_{C_i}}{\rho TTRSN_{V_i}} = \frac{TTRSN_{C_i}}{TTRSN_{V_i}} \tag{8}$$

Thus, in direct prices, k_i can be interpreted as a proxy variable to the technical composition of the capital that estimates the mechanization of sector *i*. The assumption (I) is represented with a constant k_i, implying that:

$$g_{C_i} = \frac{dC_i}{C_i} = g_{V_i} = \frac{dV_i}{V_i}, \qquad dC_i, dV_i \geq 0 \tag{9}$$

Where dC_i and dV_i respectively represent an increase in C_i and V_i caused by capitalist accumulation, and g_{Ci}, g_{Vi} are C_i and V_i's growth rate. In turn, the assumption (J) can be represented as:

$$k_1 > k_2 \tag{10}$$

With regards to ϵ_i by combining (2), (6) and (7) the following is obtained:

$$\epsilon_i = \frac{P_i}{V_i} = \frac{\rho TTRSN_{P_i}}{\rho TTRSN_{V_i}} = \frac{TTRSN_{P_i}}{TTRSN_{V_i}} \tag{11}$$

Thus, in direct prices ϵ_i is equal to an operating fee as it reflects the relationship between surplus labor $TTRSN_{Pi}$ is and the labor needed $TTRSN_{Vi}$ to cover salaries. The assumption (K) implies:

$$\epsilon_1 = \epsilon_2 = \epsilon \tag{12}$$

Where ϵ is the average operating fee. In the case of *x* using (4) and (6) the following is obtained:

$$x = \frac{V_1}{V_2} = \frac{wL_1}{wL_2} = \frac{L_1}{L_2} \tag{13}$$

Thus, the assumption of a homogenous salary (L) causes *x* to be equivalent to the sectoral distribution of employment, which complies with:

$$\frac{dx}{x} = g_x = g_{V_1} - g_{V_2} = g_{L_1} - g_{L_2} \tag{14}$$

Where g_x, g_{L1}, g_{L2} are the growth rates of *x* L_1 and L_2 respectively. For π_i, using (1) (2) and (3) results in:

$$\pi_i = \frac{\epsilon_i}{k_i} \tag{15}$$

Rewriting (15) with (10) and (12) shows that the profit rate for sector 1 (which is more modernized) is lower than that in sector 2 (less modernized):

(16)

$$k_1 > k_2 \;\rightarrow\; \pi_1 = \frac{\epsilon}{k_1} < \pi_2 = \frac{\epsilon}{k_2}$$

In other words, when direct prices govern and labor exploitation is the same in all sectors, the sector that is most modernized will obtain the lowest profit rate. Such differences in profit rates could induce capitalists from the less profitable sector to move their capital to the more profitable sector. However, this possibility can be dismissed for now as assumption (O) does not allow for intersectoral capital flow.

Regarding the structures for sectoral offer and demand I propose that income Y_i that capitalists form the *i* sector obtain by selling their goods is used to cover costs ($h_i C_i + V_i$) and profit (P_i):

$$Y_i = h_i C_i + V_i + P_i \tag{17}$$

Where h_i represents the depreciation rate of constant capital (which only includes fixed capital valued at current prices). Companies sell their goods to workers and capitalists to obtain income, according to each class' effective market demand. Thus, on the means of production market, effective demand is represented by money D_1 that capitalists spend to replenish and expand their means of production.

$$D_1 = h_1 C_1 + h_2 C_2 + dC_1 + dC_2 \tag{18}$$

In contrast, in the means of consumption market, effective demand is represented by money D_2 and groups together capitalists and workers' consumption expenditure:

$$D_2 = c_1 P_1 + c_2 P_2 + V_1 + V_2 \tag{19}$$

Where c_1, c_2 are the capitalists' consumption rates in sectors 1 and 2 respectively, and $s_1 = 1 - c_1, s_2 = 1 - c_2$ are their saving rates. If it is thought that there is general balance in all the commodities markets, then:

$$Y_i = D_i \tag{20}$$

By putting (17), (18), and (20) balance is obtained in extended reproduction for the means of production market:

$$V_1 + P_1 = h_2 C_2 + dC_1 + dC_2 \tag{21}$$

Putting (17), (19), and (20) together creates equilibrium in extended reproduction for the means of consumption market (Marx, 2009b pp, 617-620)

$$h_2 C_2 + s_2 P_2 = V_1 + c_1 P_1 \tag{22}$$

And with (21) and (22) a generalized condition of balance is obtained in extended reproduction.

$$s_1 P_1 + s_2 P_2 = dC_1 + dC_2 \tag{23}$$

In (23) capitalist saving ($S_1 P_1$ + $S_2 P_2$) should be equal to the accumulation of constant capital (dC_1 + dC_2); the fact that these are equal is equivalent to the macroeconomic identity between saving and investment, in a closed economy with no government.

Using (21) with (1), (2) and (4) and using the initial assumptions the following is obtained:

$$x^{MP} = \frac{k_2\left(h_2 + g_{V_2}\right)}{1 + \epsilon - k_1 g_{V_1}} \tag{24}$$

Where x^{MP} the sectoral employment distribution that balances the means of production market. (24) is another version of condition (22) and coincides with expression (14a) found in Harris's work (1972, p. 512) and brings greater insight into crisis caused by sectoral imbalance. For the means of production market to achieve balance, it is necessary for $x = x^{MP}$, an equality that is dependent on modernization k_i, labor exploitation (ϵ) and the growth of variable capital (g_{Vi}) which can be difficult to achieve as k_i, ϵ, g_{Vi} could respond to economic processes outside the labor market structure, which limits the values of *x*. Therefore, imbalance is very probable.

Furthermore, using balance condition (22) with (1), (2) and (4) and the initial assumptions, another version of this condition is obtained:

$$x^{MC} = \frac{h_2 k_2 + s_2 \epsilon}{1 + (1 - s_1)\epsilon} \tag{25}$$

This is also complex because to achieve balance in the market of means of consumption, it is necessary that $x = x^{MC}$, an equality that depends on the same factors as (24), plus capitalist saving decisions (s_i).

Also, given condition (23), it is important to note that the assumption of the impossibility of moving capital (O) can be represented by the following inversion function:

$$s_i P_i = dC_i \quad (26)$$

Where s_i represents an investment rate that capitalists from sector i apply to profit and allocate to accumulating constant capital, but only within their own sector, similar to ideas proposed by Harris (1972, p 511-512). Moreover, combining (26) with (1), (2), (9) and (14) and adding the assumption that x is a constant ($g_x = 0$) we arrive at:

$$\frac{s_1}{s_2} = \frac{\kappa_1}{k_2} \quad (27)$$

Thus, the impossibility of moving capital causes saving rates to comply with being equal (27) to maintain sectoral balance, whilst keeping mechanization at a constant. This is difficult to achieve if capitalists' decisions about s_i are guided by exogenous factors. In fact, (27) is similar to (16), by Harris (1972, p.512).

Likewise, (27) is helpful in interpreting Morishima's description (1973, p.118) of the inverse function used by Marx (2009b) in his schemes. This description includes another assumption: capitalists from sector 2 adjust s_2 endogenously so that no sectoral imbalance exists, whilst capitalists from sector 1 impose s_1 autonomously, (see Luxemburg, 2015). Using this premise s_2 can be resolved from (27) replacing it in (25) and obtaining:

$$x^{MC} = \frac{k_2(h_2 + s_1\epsilon)}{k_1[1 + (1 - s_1)\epsilon]} \quad (28)$$

The variant presented in (28) illustrates the asymmetric hypothesis, according to which capitalists from sector 1 take the decision-making initiative with regards to accumulation (see Ferrer Ramírez, 2009). As the present study does not need to look further at this hypothesis, the assumption that s_1 and s_2 are exogenous and condition (25) is maintained instead of (28).

All the results obtained for sectoral equality apply in reproduction on an extended scale where $S_i > 0$ and $dC_i > 0$. However, to obtain the conditions needed for equality in simple reproduction, one only needs to posit $S_i = dC_i = 0$ in (21) or (22) reducing both to the expression (29) which is identical to the condition identified by Marx (2009b, p.487):

$$V_1 + P_1 = C_2 \quad (29)$$

If $S_i = dC_i = 0$ is applied to (25) the sectoral distribution of employment that maintains balance in simple reproduction is obtained, given that expression (30) is almost identical to condition (8a) in Harris (1972, p.510)

$$x = \frac{k_2 h_2}{1+\epsilon} \quad (30)$$

In addition to highlighting the complexity of sectoral balance, the reproduction schemes presented here also formalize some Marxist thoughts on crises associated with the functions of money in circulation and payments. For example, let us look at the following idea:

> The general possibility of crisis [occurs] in the measure that money functions as a means of circulation [and] if at the time of selling merchandise it is not worth its actual value, at the moment money functions as a measure of value and, therefore, of mutual obligations, it will not be possible to fulfil the obligation with the merchandise's sale price and, therefore, neither will it be possible to pay off the entire series of transactions that retroactively depend on this one. If the merchandise cannot be sold in a determined time, even though its value has not changed, money will not be able to function as a form of payment, as it must function in pre-established periods of time. However, as the same amount of money functions here for a series of mutual transactions and obligations, it will manifest [a state of] insolvency, not only at one point, but at many points, and therefore, a crisis is caused (Marx, 1980, p.473)

The complexity of this "series of mutual transactions and obligations", associated with the circulation of money in capitalist reproduction, can be interpreted in Markovian terms. Leontief and Brody (1993) support this proposal with their input-output study of monetary flow, using Markovian chains. Their ergodic state is taken as an equivalent of economic balance, which even allows for the notion or "speed of circulation of money" to be reevaluated. Therefore, it seems pertinent to apply the same Markovian principle, but using monetary flows associated with simple and extended reproduction.

A Markovian interpretation of simple reproduction

Expressions (17) to (20) define the sectoral balances, and with $S_i = dC_i = 0$ allow us to obtain the following system of equations:

$$
RKS \begin{cases} Y_i = h_i C_i + V_i + P_i & (17') \\[2ex] h_1 C_1 + h_2 C_2 = D_1 & (18') \\[2ex] P_1 + P_2 + V_1 + V_2 = D_2 & (19') \\[2ex] D_i = Y_i & (20') \end{cases}
$$

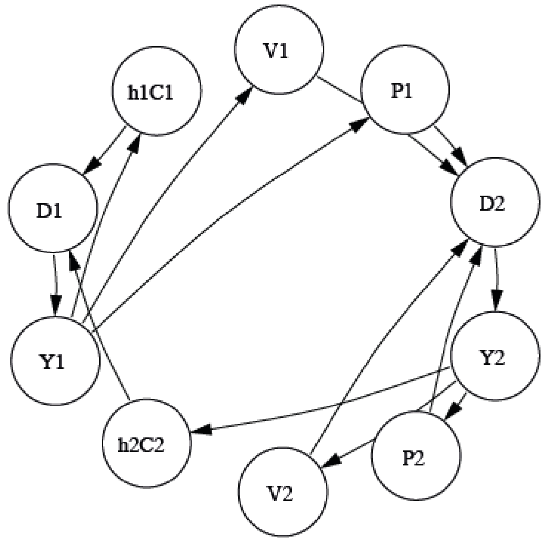
The left side of each equation of the (*RKS*) system shows monetary flow's origin, and the right side its destination. For example, (17') suggests that the money that originally comes in the form of income Y_i obtained by capitalists from sector *i* on selling their merchandise becomes cost and profit associated with depreciation of means of production ($Y_i \rightarrow h_i C_i$), salaries ($Y_i \rightarrow V_i$) and earnings ($Y_i \rightarrow P_i$). (18') indicates that the money representing costs of means of production $h_i C_i$ changes to effective demand in the means of production market ($h_i C_i \rightarrow D_i$), (19') suggests that the money represented by salaries V_i and earnings P_i changes to the form of effective demand in the means of consumption market ($V_i \rightarrow D_2$, $P_i \rightarrow D_2$), and (20') suggests that the money that was originally effective demand becomes the capitalists' income ($D_i \rightarrow Y_i$). Each of these monetary flows can be taken as directed links in a chain with nodes representing the different manifestations of money in simple reproduction (C_i , V_i , P_i , D_i , Y_i) (see figure 1). Likewise, according to Leontief and Brody (1993), the monetary flow chain in simple capitalist reproduction can be interpreted as a Markov chain, where money flow is guided according to the monetary transition matrix M^{RKS} in table 1.

Table 1. Monetary transition matrix in simple capitalist reproduction (M^{RKS})

<table>
<tr><th></th><th>h_1C_1</th><th>h_2C_2</th><th>V_1</th><th>V_2</th><th>P_1</th><th>P_2</th><th>D_1</th><th>D_2</th><th>Y_1</th><th>Y_2</th></tr>
<tr><td>h_1C_1</td><td colspan="6" rowspan="8">$0_{8\times6}$</td><td>1</td><td>0</td><td colspan="2" rowspan="6">$0_{6\times2}$</td></tr>
<tr><td>h_2C_2</td><td>1</td><td>0</td></tr>
<tr><td>V_1</td><td>0</td><td>1</td></tr>
<tr><td>V_2</td><td>0</td><td>1</td></tr>
<tr><td>P_1</td><td>0</td><td>1</td></tr>
<tr><td>P_2</td><td>0</td><td>1</td></tr>
<tr><td>D_1</td><td>0</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td>D_2</td><td>0</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td>Y_1</td><td>$\alpha_{h_1C_1}$</td><td>0</td><td>α_{V_1}</td><td>0</td><td>α_{P_1}</td><td>0</td><td colspan="4" rowspan="2">$0_{2\times4}$</td></tr>
<tr><td>Y_2</td><td>0</td><td>$\alpha_{h_2C_2}$</td><td>0</td><td>α_{V_2}</td><td>0</td><td>α_{P_2}</td></tr>
</table>

Source: compiled by authors.

Figure 1. Network of monetary flow in simple capitalist reproduction



Source: compiled by authors

Where:

$$\alpha_{h_iC_i} = \frac{h_iC_i}{Y_i} = \frac{h_ik_i}{h_ik_i + w + \epsilon} \tag{31}$$

$$\alpha_{V_i} = \frac{V_i}{Y_i} = \frac{1}{h_ik_i + w + \epsilon} \tag{32}$$

$$\alpha_{P_i} = \frac{P_i}{Y_i} = \frac{\epsilon}{h_ik_i + w + \epsilon} \tag{33}$$

That is to say, $\alpha_{h_iC_i}, \alpha_{V_i}$ show the proportion of income Y_i allocated to cover costs associated to each type of capital, α_{P_i} is the proportion of income that covers profit $(\alpha_{P_i} = 1 - \alpha_{C_i} - \alpha_{V_i})$, and $0_{A\times B}$ is a zero matrix of the $A \times B$ dimension. Note that, $M^{RKS} = \left[m_{d,d'}^{RKS}\right]$ is a Markov matrix as all the elements comply with $m_{d,d'}^{RKS} \geq 0$ and $\sum_{d'} m_{d,d'}^{RKS} = 1$. As $\alpha_{h_iC_i}, \alpha_{V_i}, \alpha_{P_i}$ are constants (and depend on h_i, ϵ, k_i, constants for initial assumptions) the conditions are met for estimating the probability that a monetary unit flowing in the chain represented by M^{RKS} will take a determined form in the long run. This can be addressed by obtaining the vector π^{RKS} (column) of stationary probability distribution of the Markov chain associated to M^{RKS}, defined by:

$$(\pi^{RKS})^T M^{RKS} = (\pi^{RKS})^T, \qquad \sum_d \pi_d^{RKS} = 1, \qquad \pi_d^{RKS} \geq 0 \tag{34}$$

Using linear algebra (34) equals $(M^{RKS})^T \pi^{RKS} = \pi^{RKS}$ where π^{RKS} is the eigenvector associated to the maximum eigenvalue of the transposed M^{RKS} (which equals one as it is a Markov matrix). On resolving (34) the stationary probabilities associated with each manifestation of money in simple reproduction obtained are:

(35)

(36)

(37)

(38)

Where and x is given by condition (29). Therefore, in simple reproduction, with the commodities markets in balance, the probability that a monetary unit will manifest in the long term as the depreciation of means of production (h_iC_i), salaries (V_i), profit (P_i), income (Y_i) or effective demand (D_i) is equal to the participation of each of these forms in total income (Y_1 + Y_2) divided by 3, which is the number of forms that money takes in simple reproduction (costs/profit, income and demand).

A Markovian interpretation of extended reproduction

Analogous to simple reproduction, the expressions (17) to (20) and (23) can build a new system of equations that describe monetary flow in extended reproduction:

Where:

(39)

This is the total accumulation in constant capital. As this system uses condition (23) where all savings are equal to the total investment - without restrictions on savings from one sector financing only their own accumulation - the interpretation is valid both for cases with or without intersectoral competition. With the (*RKA*) system it is possible to build a chain of monetary circulation in extended capitalist reproduction (see figure 2) with nodes and links that are built according to the same logic as found in simple reproduction. Equally, using the *(RKA)* system it is possible to interpret the chain as a Markov chain, where monetary circulation is guided according to the transition matrix M^{RKA} in table 2.

Figure 2: Monetary circulation chain in extended capitalist reproduction

Source: compiled by authors.

Where:

(40)

Given that the initial assumptions caused the terms α_j to be constant, M^{RKA} can be used to obtain the stationary distribution vector π^{RKA}, defined as:

(41)

On solving (41) the following stationary probabilities are generated:

Where and *x* are given by condition (25). Therefore, in extended reproduction and balance in the commodities markets, the probability that in the long-term a monetary unit manifests in the depreciation forms (h_iC_i), salaries (V_i), capitalist consumption (c_iP_i), capitalist savings (s_iP_i), income (Y_i), effective demand (D_i) or accumulation of constant capital (dC), is equal to the participation that each one of these has in the total of money manifesting in the economy, and that includes the three manifestations found in simple reproduction (costs/profit, income and demand) reflected in , plus the manifestation associated to the total accumulation of capital represented by dC. The fact that the stationary probabilities are expressed in reference to provides a useful element for the outline given in the next section.

The role of intersectoral competition in capitalist reproduction

A possible general outline

Various authors (e.g., Nikaido, 1983 and 1985 p.198; Shaikh, 2016 p.264-265; Tsoulfidis and Tsaliki, 2019 p. 255-260) propose, from a classical-Marxist point of view, that the difference between profit margins leads to intersectoral competition. Shaikh (2016, p.264) suggests that each "new investment flows more quickly towards industries with higher rates of profit". This in turn causes a greater concentration of capital in these industries and an offer that outstrips demand, which "pushes prices and profits down" (the opposite can be seen in industries with low profit rates). Thus, the intersectoral competition that comes with the movement of capital leads to a price adjustment between sectors, that in the long run can affect production prices and put sectoral profits at parity (Marx, 2009c; Shaikh, 1977 and 2016). However, this tendency can also be distorted, or even more complex tendencies may arise, generating a heterogeneity between sectoral rates of return according to the concrete form that real capitalist competition takes, as suggested by Semmler (1981 and 1984), Botwinick (2017) or even Shaikh himself (2016).

Therefore, including intersectoral competition in this model is justified, as the initial assumptions caused capitalists from sector 1 to obtain a lower rate of profit– on direct prices- than the capitalists from sector 2, as shown by (16). Thus, the impact of the technical difference on sectoral profits creates incentives for new capitalist investments from both sectors to take part in intersectoral competition. This is included in the model, replacing the initial assumptions (N) and (O) for the following. (N') all monetary aggregates are measured in prices that guarantee sectoral balance, which are different to the direct prices created according to the effects of intersectoral competition. (O') The free flow of capital between sectors is allowed, so that the inversion functions represented in (26) no longer apply. Likewise, a new assumption is added. (P) There is only one means of production, and one means of consumption. Equally, the following expressions are included:

Where p_i, Q_i, A_i each represent price, the quantity of merchandise produced and the quantity of means of production used in sector *i;* a_i is a modernization rate similar to the capital's technical composition; q_i is productivity per hour worked; *M* represents the total money manifested in the economy. According to assumption (I) it is assumed that a_i, q_i, constants. For assumption (J) it is assumed $a_1 > a_2$. Assumption (F) proposes that money is always available for circulation, and it is assumed that the monetary expression r is constant, and that money that manifests in circulation will always maintain the same proportion with respect to hours worked $L_1 + L_2$. Expression (53) is noteworthy, as it assumes that all the money that is manifested in the economy will collect three times the total income (three manifestations: costs/profits, income, and demand), plus capitalist accumulation. Thus, (53) proposes that the money that represents the value created by the labor force must consider the entire process of circulation that leads money to take on its multiple manifestations *M.* In fact (53) picks up precisely the standard of comparison obtained from the stationary probabilities which come from the Markovian interpretation of extended reproduction, as indicated by the denominators (42) to (47).

On putting (48) to (51) with (17) to (23) and (3) and (4) a new condition of balance in extended reproduction can be seen, similar to conditions (24) and (25):

(54)

Thus, the sectoral distribution of labor (*x*)must comply with equality (54) to maintain sectoral balance when the technical conditions (a_i, q_i, h_i), decisions on saving (s_i) and profit rates (π_i) are given. It is important to note that there is nothing in (54) to stop the use of measured profit rates on direct prices or, even, heterogenous profit rates measured on market prices, because the expression is obtained from generic conditions of balance. Moreover, if (48) to (51) are used with (17) (3) and (4), expression for the prices of commodities sold by each sector can be obtained:

(55)

(56)

Using (48) to (53) and (23) and (39) the following results are obtained:

(40)

Thus, (54) to (57) create a system of four equations with five unknowns that can be understood as a general outline with a degree of liberty.

The intersectoral competition dynamic

To "close" system (54) to (57) the intersectoral competition dynamic provoked by capital flow in the assumption can be included (O'). For example, the following inversion functions can be considered:[9]

(58)

(59)

Where g_{Ai} represents the growth rate of the means of production used in sector *i.* Functions (58) and (59) suggest that capitalists from each sector tend to increase their means of production ("real accumulation") at an autonomous rate , identical for both sectors and altered according to the sectoral differences between profit rates: where $\pi_i > \pi_j$ capitalists from sector *j* will accumulate fewer means of production in their own sector and will move capital to another sector, all the while respecting the macro equilibrium between saving and investment shown in (23). Deducting (58) and (59) the following is obtained:

(60)

Expression (60) is identical to equation (5) suggested by Dutt (1988, p.141) (see also Dutt, 1977, p.477) to study intersectoral movement of capital. As there is not technical change ($a_i = A_i / L_i$ constant), then:

(61)

Combining (61) with (60), (13) and (14) we obtain:

(62)

Where (62) shows that the growth rate of *x* changes because of the movement of capital motivated by the differentials between rates of profit. Thus, (62) together with (54) to (57) complete a system with five equations with five unknowns that, on being simplified, can be summarized with the following differential equation:

(63)

Where h_i, a_i, s_i, q_i, r, ρ, w are constant parameters. Assuming a simplified case (Q) where there is no depreciation $h_i = 0$, all profits are saved ($s_i = 1$) and the monetary expression of work is standardized ($\rho = 1$) a compact version of (63) with stable behavior and a positive equilibrium value of x^* can be proposed (see figure 3).

Figure 3. A simplified dynamic of sectoral employment distribution

Source: compiled by authors.

(64)

To find x^*, note that, in the long-term, intersectoral competition is equal to the sectoral rates of return with an average rate of π^*:

(65)

Replacing (65) in (54) to (57) we obtain a four-equation system with four unknowns, that guarantee balance in long-term extended reproduction considering an intersectoral competition that generates an average profit rate of π^* and production prices, :

Using the simplified case (Q) again, the following solutions are obtained to system (66) to (69):

Where:

(74)

In this case, other interpretations arise. For example, (70) shows that the more modernized sector 1 is over sector 2 , the lesser the sectoral distribution of balance of employment will be , which means that sector 1 has less relative weight in total employment. Likewise, so that $x^* > 0$ in the long-term, it is necessary that $w < 1/3$, or there would be no x^* to, simultaneously, guarantee sectoral balance and stability, a limitation that could be considered another manifestation of a possible crisis due to sectoral imbalance. Thus, it can be seen that employment distribution – and its dynamic- continues to be conditioned if imbalance is to be avoided.

In this context, the effect of nominal salaries on each of the model's solutions within the range of positive values in the long term is emphasized (see figure 4). Returning to (70) we can see that increasing the salary in sector 2 tends to absorb relatively more employment than in sector 1 , which makes sense given that a higher salary means a greater effective demand of means of consumption, produced by sector 2. (71) also suggests a possible non-monotonous link between salary and sector 1's cost of production. However, (72) shows that a higher salary always increases production costs in sector 2. A significant relationship can be seen in (73) where there is an inverse connection between the average profit rate and salary , similar to the results Sraffa (1960, p.25 and Morishima (1973, p. 64) obtained, but with the difference that this inverse relationship comes from a Marxist perspective, where the monetary expression for work ρ, is defined using the results of the Markovian interpretation of extended reproduction.

Figure 4. The effect of salaries on jobs, production costs and the average rate of earnings

Note: numerical simulation obtained with

Source: compiled by authors.

The complexity of long-term equilibrium

A point that I wish to highlight with the model presented in this article is how complex it can be for capitalism to achieve stability and sectoral equilibrium in the long term, if the true magnitude of intersectoral competition is considered. The very fact that sectoral distribution of long-term employment must be x^* and that its dynamic is limited by (63) may pose serious proportionality problems if we consider that the participation of each productive sector in employment can depend on the "capitalist development" of each society (as well as factors such as being placed within international trade as a central or peripheral- dependent capitalist country). Likewise, the structural differences between sectors 1 and 2 can cause x to change to x^* even in the long term, provoking persistent differences between sectoral profit rates and thus impeding rising production costs and only one average profit rate. In fact, Semmler (1981, p 41-42;1984) states that Marxist theory on competition allows for long-term differentials in profit rates due to factors such as: imbalances between supply and demand and long capital circulation times; restrictions to production conditions, and limitations in the movement of capital; high productivity in some businesses in one sector that is not widespread in other companies. To illustrate this possibility, investment functions (58) and (59) can be replaced with the following:

(58')

(59')

Where the following assumptions are added. (R) The autonomous increase of means of production installed in sector 1, is slower than in sector 2 . This could be due, for example, to sector 1 requiring larger and more sophisticated infrastructure than sector 2, and consequently having a longer period of installation and taking longer to be fully operational. (S) There are barriers that limit the access to capital for sector 2 because it faces higher risk as it depends on the workers' limited capacity for consumption. If (58) and (59) are deducted and the other model assumptions are applied, the results are:

(62')

(62) is similar to equation (8.55) proposed by Dutt (1990, p.180) to represent a persistent difference in the long-term profit rates.[10] If (62) is replaced by (62') and is put together with (54) to (57) a system of five equations with five unknowns is obtained, an alternative to the system presented in the subsection on intersectoral competition dynamics. On resolving this system, a new differential equation is obtained that defines the sectoral employment distribution dynamic *x*, but which now depends on the additional parameters :

(63')

These new parameters lead to the solution of the new system of equations to cause the long-term sectoral profits to not be equal to an average rate, but to differ according to the following expression:

(65')

As and . cannot be defined in advance if or vice versa. What can be said is that . Equally, the sectoral distribution of employment *x* will not coincide with *x** , even in the long term. Thus, in accordance with Semmler (1981), it can be said that the differences between profit rates- and the actual value to which x converges – will depend, in the long term, on the intersectoral competition's specific conditions (represented by). These additional considerations add even more complexity to the possibility of sectoral balance in a capitalist economy and its convergence to homogenous "centers of gravity".

4. CONCLUSIONS

Marx's schemes for simple and extended reproduction have left a mark on economic thinking, as well as aiding the understanding of how complex maintaining sectoral equilibrium is. With that in mind, this article has revisited capitalist reproduction from a model that combines three elements: the identification of conditions for equilibrium for the markets of means of production and consumption, a proposal of monetary flow networks that allow capitalist reproduction to be interpreted from a Markovian perspective, and the role of intersectoral competition and its complexities.

The model's results include identifying several limitations of distribution of employment to ensure sectoral equilibrium, following, and extending ideas suggested by Harris (1972). Then, using Markov chains, a pattern emerged in the stationary probabilities of monetary flow in extended reproduction that is useful in reinterpreting the monetary expression attributed to time worked. In so far as intersectoral competition, a system of equations was created that describes the movement of capital reacting to differentials in the rates of profit, as well as a collection of solutions for long term balance for the sectoral distribution of employment, sectoral production costs and the average rate of profit. These solutions then gave rise to an interpretation – both preliminary and simplified – of the effect of salaries, highlighting an inverse relationship between the average rate of profit and salaries. Additionally, a brief variant of intersectoral competition was created that allows long-term differentials in rates of profit, adding even greater complexity to the possibility of sectoral balance.

This article's greatest contribution may be that it provides a general outline of capitalist reproduction that can be taken further and in different directions. Thus, interpretations in more complex chains with additional sectors, or more complicated monetary flows, can be included. Another outcome might be that the monetary transition matrix could also change with time due to capitalist accumulation, for example, because of modernization or distributional changes to the style of model proposed by Goodwin (1967). Another extension could include a more refined definition of the monetary expression of time worked testing different interpretations available in literature (see Moseley, 2016). It would also be interesting to extend the implications of the skewness hypothesis represented in expression (28). Likewise, sectoral rates for the use of installed capacity could be included. These would be defined in the short term, prior to the growth in the average profit rate (see Dutt, 1990). Finally, there is additional skewness in the flow of capital, according to ideas put forward by Semmler (1981 and 1984), Botwinick (2017) and others.

These thoughts and others will certainly reinforce- or refute- several of Marx's ideas on the level of complexity in capitalist reproduction. Studying such complexity is important if one desires to have better tools with which to understand and tackle the crisis associated with the "chaos" of competition based more in financial gain than social stability.

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[1] Crisis can be generically understood as the "temporary interruption" of capitalist reproduction (Marx, 2010, p.134; Shaikh, 1978, p.49)

[2] See Desai (2019) for a summary of the history of economic Marxism.

[3] See Rodolsky (1977, chapter 30) for a review of other early commentators.

[4] There were earlier relevant mathematical reinterpretations, but these were less known. For example, Bronfenbrenner (1966) used the schemes to talk about crisis from "liquidation" and "realization"

[5] See Ithoh (2020) for a summary of computational methods in Marxist economy

[6] See Cogliano *et al.* (2020) for a review of computational methods in Marxist economy.

[7] For simplicity Marx's original assumption of salaries paid at the beginning of production

[8] These financial sectors avoid the need for a central agent acting as financial provider (e.g., government). This option is analyzed by Trigg (2006, p.53). In turn, two different, specialized financial sectors are adopted to avoid distortions when distributing sectoral gains between productive gains and interest.

[9] The inclusion of the rate of return in (58) and (59) is inspired in Robinson (1956 and 1962. For other alternatives, see Araujo and Teixera (2015). Two inversion functions are used, as capitalists "do many things as a class, but they certainly do not invest as a class", Kalecki 1967, p.455)

[10] Dutt (1990) proposes his equation for North-South models where the greatest risk is associated with the South.