Dynamic input-output model for a small economy

Noah Aron Fuentes[a] and Sarah Martínez Pellégrini[a]

[a] The College of the Northern Border, Mexico.

Email addresses: afuentes@colef.mx and sarahm@colef.mx respectively.

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Abstract

This article applies the Dynamic Simulation (DS)technique to the dynamic input-output model (DS/IO) designed by Johnson (1985 and 1986). The model is a generalization of the static input-output model (Leontief, 1941), with an additional emphasis on economic imbalance. The DS/IO consists of a system of 4n differential equations and 2n constraints. Therefore, attempts to model analytically are likely to be unsuccessful without numerical simulation. The DS/IO used data from the Mexican economy for calibration and validation. It was found that the dynamic multipliers are greater than the static multipliers due to the accelerator effect. The simulation of the DS/IO is also validated by the fact that the adjustment between current values and production projections from 2013 to 2019 is plausible.

Keywords: dynamic input-output model; dynamic systems; simulation; Mexico.

1. INTRODUCTION

The works of Wheat and Pawluczuk (2014), Jackson *et al.,* (2016) and Cordier *et al*. (2017) integrate the input-output model (IOM) and dynamic systems (DS) into an ecological-economic system (EES). In such systems, the flow of material changes from one static state to another depending on the difference between its current level and its level of equilibrium. In a EES complexity is an essential factor (Limburg, 2002; Cordier *et al*., 2017); otherwise, there is a major failing upon not incorporating real functional processes, which can lead to failures in public policy (Constance, 1987) when these simulations are used. There are at least two main sources of complexity: the first, the relationships between system elements that are not linear, predictable, or controllable (Folke *et al*., 2002). The second, the dynamics between the elements incorporate lag, restrictions, limits and feedback loops (Dasgupta and Mäler, 2003).

Incorporating in some way the investment or capital formation process in a static IOM transforms it into a dynamic IOM. This text aims to apply the Dynamic Simulation (DS) technique developed by Forrester (1961) to the dynamic IOM (DS/IO) designed by Johnson (1985 and 1986); which is a generalization of a static IOM (Leontief, 1941) to a situation of economic imbalance.

DS/IO differs from other models in this field's literature as it simultaneously incorporates time delays and adjustment mechanisms that reflect investment or capital formation lag, resulting in an SD/IO imbalance.

The DS/IO takes into consideration the context in which there are production adjustments in the short-term due to differences between rates of change in consumption (demand) and production (supply); that is, it considers the situation in which consumption *(demand)* in year *t* always tends to induce an increase in production (supply) by *t+1,* subject to the restriction that production does not exceed productive capacity. Therefore, production and consumption are different concepts with equal values when the system is in balance, but not when it is experiencing an imbalance. That is, the difference between production and consumption is that the latter must accumulate in order to conserve physical flows by means of the production-distribution channel. This is the reason a certain inventory level is required, in order to dissociate these two different flow speeds. If there is an increase in consumption, inventories fall and production needs to increase more than an equal amount than that of consumption, thus restoring the balance between inventories and consumption.[1]

Similarly, the DS/IO takes into consideration the situation in which capital formation depends on installed production capacity *t* and operated in period *t+1* subject to the restriction that net investment is equal to or exceeds replacement investment, ensuring a non-negative gross investment.[2] Here, gross investment represents the accumulation of net investment flows (or new production capacity derived as the difference between the current and desired productive capacity), and this accumulation then influences the flows rates. If there is an excessive accumulation of gross investment it will not foster successive investments, which will affect new productive capacity and, in turn, production. Finally, the model has the specification of desired productive capacity which reflects the dynamic impact a variance in consumption (demand) has on system behavior and generates an investment flow lag scheme to describe the actual processes of capital formation up to their obsolescence.

The DS/IO is more complete than the static IOM from the point of view of incorporating real operation processes, allowing the projection of time paths for production stocks and *n* sectors into the future while taking into consideration not only the growth of final autonomous demand, but also the capital formation which is indispensable in making such growth possible, along with the constraints of installed productive capacity and the limits of sectoral disinvestment. The behavior of time paths is the result of applying the acceleration principle and assuming that production and investment do not instantly vary with changes in the final autonomous demand. Time lags and more flexible mechanisms tend to generate more feasible trajectories over relatively long periods of time.

DS/IO is a useful tool for exploring development issues related to the possibilities of economic growth, methods for accelerating this process and making it more efficient.

In order to calibrate it, the DS/IO has been "adjusted" with information from the 2013 Matriz Insumo Producto de México (*Mexico Input Output Matrix*) (INEGI, 2014); reducing the 72 individual branches into three groups of sectoral activity (primary, secondary and tertiary). The work carried out is related to the analysis of the dynamic impacts on the economy stemming from an increase in public spending in the secondary sector (5% of the sectoral total) which manifests itself in a two-year period.

Furthermore, DS/IO simulation is validated. Ideal validation occurs when a simulation is capable of reproducing the experiment's data or the observations considered (San Isidro, 1998). In this case, DS/IO validation involves having a small economy's (region, state, municipality or city) GDP information by type of sectoral activity and the total GDP from the cross-sectoral table, as well as GDP values by sector and total activity type for the period of 2013-2019. As the information at the subnational level is more difficult to obtain or of inferior quality, for demonstrative purposes national information is used.[3]

This text is organized as follows: the second section summarizes the fundamental conceptualization of DS/IO. The third section shows the programming of a DS/IO, highlighting the production adjustment and capital formation feedback loops. The fourth section presents the potential of DS/IO for a small economy through an analysis of sectoral dynamic multipliers resulting from a shock in final demand. Section five shows the validation of DS/IO, involving a projection in a historical period of the Mexican economy from 2013-2019. The sixth section presents the conclusions.

2. JOHNSON'S DS/IO MODEL

Johnson's DS/IO is an abstraction of a static IOM (Leontief, 1941) with an emphasis on economic system imbalance. The DS/IO does not require foregoing Leontief's proposed approach.[4] On the contrary, the dynamic and static models' conditions for a basic equilibrium are preserved, but certain relationships that describe reaction paths during the process are incorporated. This leads to the modification of the *balance equation* which relates the product level to supply and demand conditions, the level of gross capital formation that changes with production capacity, the response of installed production capacity to the differences between its current and desired values, and the changes in the desired production capacity in response to changes in demand. Furthermore, it includes a limitation on installed production capacity and limits disinvestment in each sector. The complete DS/IO is as follows:

$$\dot{X(t)} = \hat{\varphi}\,[AX(t) + Y(t) + I(t) - X(t)] \qquad (1)$$

$$I(t) = B\,[\hat{d}X^c(t) + \dot{X^c}(t)] \qquad (2)$$

$$X^c(t) = \hat{k}\{X^{c*}(t) - X^c(t)] \qquad (3)$$

$$X^{c*}(t) = \alpha + \beta(AX(t) + I(t) + Y(t)) \qquad (4)$$

subject to the following restrictions:

$$X(t) \leq X^c(t) \qquad (5)$$

$$\dot{X^c}(t) \geq -\hat{d}X^c_t \qquad (6)$$

The imbalance characteristic is achieved by adjusting the balance equation which in turn allows short-term adjustments in production due to differences between rates of change for consumption (demand) and production (supply). The modified balance equation (1) represents the rate of adjustment of sectoral production flow per unit of time ($X^{\bullet}(t)$). The proposed adjustment will over time eliminate surplus demand ($AX(t) + Y(t) + I(t) - X(t)$). In this equation ($A$) the technical coefficients matrix, $X(t)$ is gross production, $Y(t)$ is the final demand excluding gross investment ($I(t)$) and $\hat{\varphi} > 0$ is a diagonal matrix of the constant accelerator in the economy, which shows the speed of production's response to changes in demand.

For those sectors that are limited by a productive capacity rate, the restriction of equation (5) is added. That is to say that one assumes that production should not exceed production capacity.

Equation (2) represents the gross capital formation process. Gross investment $I(t)$ can be disaggregated in capital replacement investment (depreciation) with $X^c(t)$ representing total productive capacity and insert equation is the vector for capital replacement investment demand. The same is true in net (or induced) investment with $B(X^{c\bullet}(t))$ representing the vector of investment demands for the formation of new capital; while B is a distribution matrix of investment demands.

For those sectors whose new productive capacity (net investment) is period *t*, and this new capacity is installed and operated in period *t + 1*, one must add the restriction of equation (6) which states that net investment must be equal to or exceed replacement investment, ensuring a non-negative gross investment.

Equation (3) represents net investment and considers that its adjustment depends on the difference between the desired ($X^{c^*}(t)$) and current ($X^c(t)$) capacity where $\hat{k}$ is a diagonal matrix of marginal capital-product (accelerator) relationships.

Equation (4) proposes that the desired productive capacity ($X^{c^*}(t)$) at any point in time is a linear function of consumption (demand) over time, with α an intercept vector that represents a desirable state of surplus installed capacity and β is a diagonal matrix of slopes that measures the desired proportion of installed capacity.

In short, DS/IO can be described as a system of *4n* differential equations with *4n* endogenous variables $X_i(t)$, $I_i(t)$, $X^c(t)$, and $X^{c^*}(t)$; the following exogenous variables and parameters Y(*t*), *A*, *B*, *Φ*, *k*, *d*, *α*, *β* and *t* which has *2n* restrictions $X(t) \leq X^c(t)$ y $X^{c^*}(t) \geq -\hat{d}X^c(t)$. Due to its complexity, it is impractical to try to achieve an analytical solution, but a numerical simulation using the dynamic simulation technique can be applied.

3. SD/IO MODEL INTEGRATION WITH DS

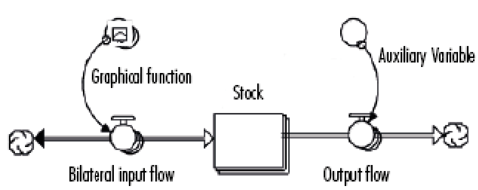
Dynamic models require some mathematical handling to find the analytical solution to difference and differential equations and have a severe limitation in the number of dynamic variables that can be calculated.

The DS technique developed by Forrester (1961) makes it possible to break down both barriers, allowing one to develop dynamic models without the need to handle complicated analytical difference or differential equations and, above all, allows them to be calculated with an unlimited number of variables. In this sense, DS has been a spectacular step forward in the use of dynamic models.

Stella software (9.1.5) was used in order to integrate the DS analysis method with DS/IO. The software is a visual modeling and simulation language for DS which uses the symbology of the Forrester diagram.

The basic diagram is presented in Figure 1 (Cervantes *et al.*, 2007).[5]

Figure 1. Forrester diagram



Source: Created by the author based on Cervantes et al., 2007.

The icons used in building the model in this figure are as follows:

The *stock,* represented by the rectangle, corresponds to variables whose evolution is significant for the study of the system; they accumulate material through the channels controlled by the valves.

Output and input flows define the behavior of the system, as they determine the speed of the flow of matter (through the material channels) according to a set of associated equations, which depend on the information that the valves receive from the system (levels, auxiliary variables and parameters) and the environment (exogenous variables). They're represented as a valve.

Arrows represent the internal relationships between these variables. The direction of the arrow indicates the causal flow.

The bilateral output and input flows are rates that can take on positive and negative values and are represented by a double-arrow material channel.

The converter or auxiliary variables correspond to intermediate steps in the calculation of functions associated with the valves; they are used to simplifying the process as certain mathematical calculations are used in various equations. They are represented by a circle and, when they are matrices, by a shaded circle. They are connected by arrows.

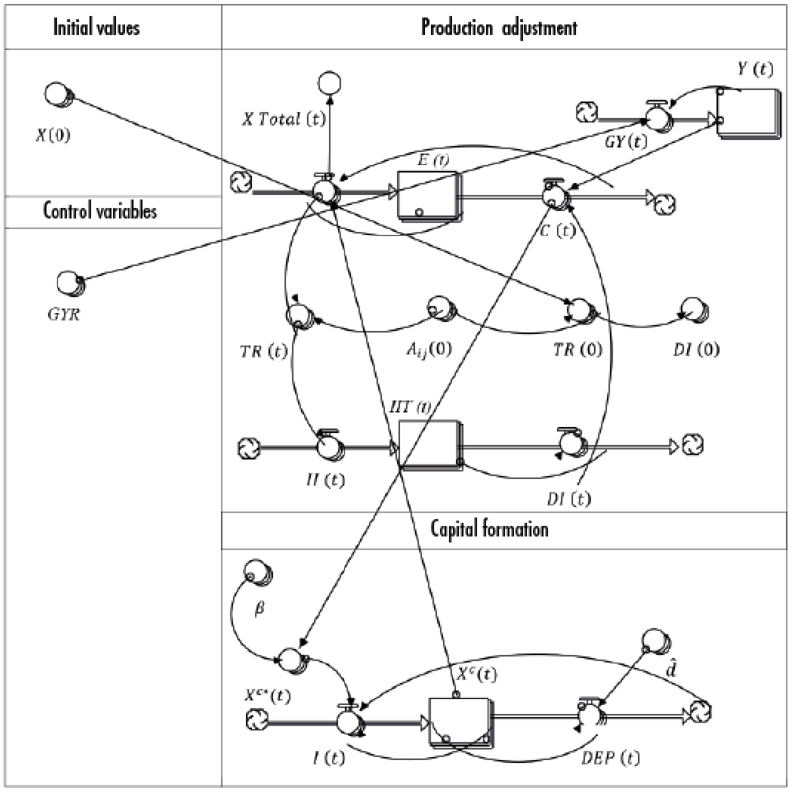
The graphical function displays a dialog box that allows one to define values graphically. It is represented by a circle with an image inside.

Clouds represent sources and sinks, that is, an indeterminate (infinite) amount of material input and output.

STELLA software is therefore a graphical interface where programming is designed using objects corresponding to the DS/IO equations.

Figure 2 highlights objects that represent DS/IO programming, especially initial model values, control variables, major relationships, and feedback loops.

Figure 2. DS/IO model and its feedback loops



Source: Created by the author using STELLA software.

We will now define inventory stock In an EES system, production and consumption move towards equilibrium at a rate that depends on the difference between demand and supply, meaning the economy responds to the change in inventory (Xu, 2014; Yamagushi, 2002).

In DS, inventories are physical stock which captures accumulation levels and decreases when there is a rise in consumption, inducing a production response equal to the new consumption plus the drop in inventories (conservation principle). Equation (1) can be rewritten in terms of the stock and flow equation as:

$$E_i(t) = E(0) + \int_0^t (X_i(t) - C_i(t))dt \qquad (7)$$

Where inventories are defined as $E(t) = [X(t) – C(t)]$, consumption (demand) as $C(t) = AX(t) + Y(t) + I(t)$, production $X(t)$ and i = 1 ... s (sectors).

The input flow ($X(t)$) is defined as the production levels which are equal to consumption plus inventory replenishment (consumption minus inventories). Production is twice consumption minus inventories.[6] Furthermore, this value is considered to be limited when production is less than the current installed production capacity.[7]

$$X_i(t) = min(2 * C_i - I_i, (2 * C_i - I_i) * \frac{X_i(t)}{X_i^c(t)}) \qquad (8)$$

Depending on the consumption function (demand) a stock of intermediate goods available is required to carry out production ($IIT(t)$). Based on the definition that corresponds to the demand for intermediate goods ($AX(t)$) we get the required stock of net intermediate goods for each sector in the economy. The purpose is to allow a dynamic response in the economy to production. The stock and flow equation for intermediate goods is:

$$IIT(t) = IIT(0) + \int_0^t (II(t) - DI(t))dt \qquad (9)$$

Where the intermediate goods required for the production of each industry (*II*(*t*)) are obtained by subtracting from each sector's production demand their own intermediate goods and from the rest of the intermediate demand (*DI*(*t*)). In this expression, *IIT*(*t*) is the stock of net intermediate goods from inputs and outputs, and *IIT*(0) is the stock of net intermediate inputs at the starting point.

IIT(*t*) depends on the auxiliary converter of the initial intermediate transaction matrix (*TR*(0)), which is used only to measure the total use of all initial intermediate goods computed as the multiplication of the matrix of technical coefficients at the beginning ($A_{i,\,j}$ (0)) by the vector of the total of each industry's production value in the first period (*X*(0)). Due to how STELLA calculates matrix and vector products, before creating the initial intermediate transaction matrix, data from the initial technical coefficient matrix and the initial industrial production value vector must be transposed before multiplying.[8] To measure the total use of initial intermediate goods (*DI*(0)) the sum of each row $i\,(\sum_{j=1}^{n}\;A_{i,j}(0)\cdot X(0))$ is obtained, with it representing the part of the production of sector *i* intended for intermediate consumption by other sectors, including sector *i.* The measurement of production's intermediate transaction matrix (*TR*(*t*)) is obtained as the product of the initial technical coefficient matrix ($A_{i,j}$ (0)) times the new production vector (*X*(*t*)).

Continuing the consumption function, the final demand which excludes investment (*Y*(*t*)) records transactions relating to the final use of products in the economy. Final demand is an aggregate that includes household consumption, government spending, stock variation and exports. It is assumed to be exogenous and grows at given proportional rates (*GY*(*t*)). The stock and flow equation for final demand which excludes investment can be written as:

$$Y(t) = Y(0) + \int_0^t \; GY(t)dt \qquad (10)$$

Where, *GY*(*t*) is a variable in terms of sectoral growth rates that can take on positive or negative values representing the remainder of final demand; *Y*(0) is the remainder of final demand at the initial time. The input variable is obtained as:

$$GY(t) = \; GYR * Y\,(t) \qquad (11)$$

Where *GYR*(*t*) is the growth rate of final demand at time *t.* To further clarify the equation, on the one hand, one can opt for a simplified assumption that the final exogenous residual demand from all sectors grows at a constant positive rate (3%). On the other hand, growth rates can be assumed to be differentiated according to sector (positive, negative or 0).

From this, we establish an initial feedback loop called "Production Adjustment" characterized by the fact that inventories are a physical stock that captures accumulation levels. When an increase in consumption (demand) comes to pass, it decreases and induces a response in production disproportionately greater than the increase in consumption. However, production (supply) is given the limitation that it must not exceed the productive capacity of each sector. The change in production necessitates meeting the requirements of the demand for intermediate goods and the final demand which, together, determine consumption (demand) by completing the cycle.

The set of production equations described above indicates the adjustment of production, regardless of capital formation, the reaction of installed production capacity to differences between current and desired values, or changes in desired production capacity in the face of changes in demand. However, the DS/IO indicates the existence of a relationship between capital, productive capacity, and demand: the capital-product relationship (or accelerator coefficient).

The next step is then to include gross capital formation requirements for production (Leontief, 1953). The traditional manner of doing so is by using the following equation:

$$I(t) = \; B(IN(t) + \; DEP(t)) \qquad (12)$$

Where *I*(*t*) represents gross investment demands based on their origin, *IN*(*t*) are the investment demands for the formation of new capital stock according to the destination, *DEP*(*t*) is depreciation or replacement of capital according to the destination, while *B* represents the distribution matrix of capital demands according to origin.

The accelerator principle applies only to demand for new investment (or net investment). There are different possible functions for investment. In this case the new investment depends on the changes between the industries' desired installed capacity $X^{c^*}(t)$ and current installed capacity ($X^{c}(t)$). The equation is:

$$IN(t) = \hat{k}(X^{c*}(t) - \; \dot{X}^{c}(t)) \qquad (13)$$

The accelerator $(\hat{k})$ tries to reflect the lag needed to match the desired and current capacity. The availability of capital goods inventories will affect the amount of new investment as well as the accumulation of productive capacity.

Furthermore, net investment incorporates asymmetric processes in which capital will grow or decrease, as when consumption (demand) increases, and capital accumulates the positive difference between net investment and depreciation. Meanwhile, when it decreases, capital outflow is limited to depreciation. In other words, net investment is limited to being equal to or greater than depreciation.

In order to project the requirements of replacement investment, these are assumed be a proportion of the current installed capacity.

$$DEP(t) = \hat{d}\, X^C(t)) \qquad (14)$$

Finally, the model must take capital stock into account to reflect the dynamic impact of a change in consumption (demand) on system behavior. Therefore, a behavioral relationship between desired productive capacity $X^{c^*}(t)$ and consumption (demand) needs to be established. Based on the hypothesis that at any moment in time it is a linear function of consumption (demand) in the same period of time, or:

$$X^{c*}(t) = \alpha + \hat{\beta}\,(AX(t) + Y(t) + I(t)) \qquad (15)$$

Where α is an intercept column vector (representing a "buffer" comprised of the excess of industrial installed capacity) and $\hat{\beta}$ is a diagonal matrix of slopes (it represents the desirable ratio of industrial installed capacity relative to intermediate and final demand). As such, the current installed industrial capacity is in turn a lag function of the desired industrial installed capacity.

The desired installed production capacity ($X^{c^*}(t)$) size converter is a linear function of current consumption (demand). In the beginning it behooves us to use only the slopes (intercepts are 0). The converter for the desirable ratio of industrial installed capacity to consumption or demand (β) was calculated in the same manner as those values that make the desired installed capacity equal to the initial consumption.[9] The stock of current installed productive capacity is measured in terms of production monetary units rather than investment monetary units. Initial values are not required here because the model automatically determines values close to the equilibrium of the initial production capacity of each industry. Meanwhile, the input flow of gross investment ($I(t)$) depends on the difference of the desired ($X^{c^*}(t)$) and installed ($X^c(t)$) productive capacity multiplied by the accelerator $(\hat{k})$ for which, instead of a diagonal matrix with capital-product sectoral marginal relationships, we used a constant for all sectors.[10] Finally, the depreciation output flow ($DEP(t)$) decreases the installed production capacity ($X^{c^*}(t)$). The depreciation $(\hat{d})$ projection converter was calculated as a constant proportion of current installed capacity.[11]

From the above, a second feedback loop called “Capital Formation” is established, characterized by gross investment representing the accumulation of net investment flows. This accumulation then influences the rates of flows. If there is an excessive gross investment accumulation, it will not encourage further investments (it decreases the desired capacity), which will affect the new productive capacity which in turn affects production, closing the cycle.

To close, it is important to mention that using objects for programming allows one to design a DS/IO in an easy and welcoming visual environment. Furthermore, the programming of the DS/IO was carried out in matrix form so the abstraction of the case in any number of branches of activity is only by means of appropriate change in the dimensions of the matrices involved (Johnson, 1986; Fuentes *et al*., 2015).[12]

4. DS/IO’S STATE OF EQUILIBRIUM

Finding a (long-term) static equilibrium for SD/IO which exhibits structural consistency is important due to its nature of dynamic imbalance. The static state equilibrium implies that stocks stop changing, which, to be precise, means that net flows become 0. This happens when production only meets the demand for intermediate goods plus final demand, in short, when a dynamic IOM is transformed into a static IOM.

It is of particular interest to review a DS/IO outside a static state of equilibrium. Therefore, as an experiment, a factor of imbalance is introduced by means of an exogenous change in final demand. The dynamic multipliers are then analyzed. The multiplier concept essentially shows how an exogenous increase in final demand generates an increase in, for example, production greater than the original increase (Ángeles, 1991).

Impact or multiplier analysis is one of the most important applications of a DS/IO at the subnational level (region, state, municipality or city), not only because economic activity at that level is determined primarily by the level of final demand, but also because prices, wages and the interest rate are exogenous in these small economies (Johnson, 1985 and 1986).

At this point, it is important to note that since one wants to not only perform the multiplier or impact multiplier analysis, but also validate the simulation of the DS/IO (see next section), one also needs to compute both GDP by sector activity type and total GDP based on the cross-sectoral table, as well as have GDP values by sector activity type and total GDP values for a whole historical period. This is why we chose to use national information given its statistical consistency and availability.[13]

We illustrate the multipliers or impact analysis using the 2013 Mexico Input-Output Matrix published by the National Institute of Statistics and Geography (INEGI, 2014). The entire Mexican economy is divided into 72 industrial branches. For the purposes of the analysis, this total was aggregated by activity type. Sector 1 (primary activities) corresponds to the first 10 industrial branches of the national table; Sector 2 (industrial activities) adds 51 branches and Sector 3 (services) includes the remaining industries. The aggregated cross-sectoral matrix is presented in Table 1 along with GDP data by type of sectoral activity and the country’s total GDP.

Table 1. National PIM by type of activity, 2013 (millions of pesos)

Activities	Primary activities	Industrial activities	Services	Intermediate demand	Final demand	Use total
Primary activities	64 515	421 732	5 170	491 417	288 325	779 742
Industrial activities	117 098	3 021 320	746 758	3 885 176	9 753 926	13 639 102
Services	55 740	1 853 896	1 805 456	3 715 092	9 508 711	13 223 804
National intermediate consumption	237 353	5 296 948	2 557 383	8 091 685	19 951*	27 642 648
Imports	65 594	3 200 166	633 124	3 898 884	1 384 564	5 283 449
Taxes on net subsidy products	-419	-19 679	29 556	9 459	625 109	634 567
Total uses at consumer's prices	302 529	8 477 436	3 220 064	12 000 028	21 561*	33 560 664
Total gross production	477 213	5 161 666	10 003 740	15 642 620	0	15 642 620
Producción bruta total	779 742	13 639 102	13 223 804	27 642 648	0	27 642 648
GDP	476 794	5 141 987	10 033 297	15 652 079	625 109	16 277 1

Note: * thousands.

Source: INEGI. SCNM. https://www.inegi.org.mx/temas/mip/

The DS/IO simulated a scenario of an exogenous spike in final demand of 682 billion which falls to the secondary sector (5% of its total value) and is made in the span of two years. The economy is monitored one quarter before the change, during the change and for a year and a half after it, using quarterly periods.

As for how the DS/IO programmed in STELLA is linked to EXCEL, one must remember that while all variables are captured directly in the DS software, some are processed in EXCEL for technical efficiency. In particular, DS/IO equations in STELLA are linked to the EXCEL platform in order to calculate the dynamic multipliers.

Following this procedure, multipliers can be calculated. Static ones only show the magnitude of the economic impact, while dynamic multipliers show the magnitude of the impact, the time of occurrence and the temporal composition between the sectors. The dynamic multiplier effects of the final demand shock on the secondary sector for selected quarters are presented in Table 2. These multipliers are interpreted as Type II multipliers because the model is closed off to households. Dynamic multipliers are larger than static multipliers and for the primary sector are (1.44), (1.56) for the secondary, and (1.26) for the tertiary sector with (1.42) as the average for the whole economy. This result is due to the accelerator principle which recognizes that any temporary increase in production requires a higher capital stock which in turn involves more depreciation costs.

Table 2. Semestral dynamic multipliers due to a final demand shock

Semester	Primary sector (S1)	Secondary sector (S2)	Tertiary sector (S3)	Economy Average
0.5	1.42	1.51	1.24	1.39
1	1.44	1.56	1.26	1.42
1.5	1.45	1.56	1.26	1.43
2	1.49	1.58	1.27	1.45
2.5	1.55	1.62	1.28	1.48
3	1.55	1.62	1.28	1.48
3.5	1.55	1.62	1.28	1.48

Source: DS/IO direct estimation.

One can deduce from Table 2 that the economic impact of an exogenous change in activity type in the secondary sector is greater for all periods. For example, in the first year the multiplier is 1.56, 1.58 for the second year, and 1.62 for the third. In general, the magnitude of the dynamic multiplier is positively correlated with the ratio of intermediate goods to total production. In other words, the sectoral multiplier will be higher when the ratio of intermediate goods used in total production is larger.

Table 2 also shows that the convergence rate is different depending on the nature of capital for the sector. For example, in sectors characterized by a higher proportion of capital per product, as with the secondary and tertiary, which exhibit a lower convergence rate for the multiplier. The tertiary sector has a low convergence rate with a multiplier of 2.16 in year one, 0.70 the following semester, 1.40 in year two and 2.07 in the next quarter. A pattern similar to the above occurs for the secondary sector.

To summarize, the DS/IO experiment, with regards to a final demand shock, makes it possible to verify that dynamic multipliers are greater than static multipliers, that their magnitude per sector depends on the ratio of intermediate goods to production and that their impact's duration is a function of the capital-product ratio.

5. DS/IO VALIDATION

Validation of a model occurs when the simulation is able to reproduce the experimental data or observations taken into consideration. The DS/IO should be tested using the prediction criterion of a period of time for a small economy (region, state, municipality or city). However, as previously mentioned, at the subnational level there is no official product input information, let alone a time series compatible with this accounting scheme.

This is why a DS/IO validation procedure based on the 2013 National Product Input Matrix is presented. From it, indicators are "projected" by sectoral activity group GDP and the total GDP for the period of 2013-2019. The time series of current GDP values by sector activity and the total were obtained from INEGI; it is deseasonalized and in millions of pesos in 2013. The greatest hurdle was using the cross-sectoral table to compute total GDP and by sectoral activity type. Both datums are calculated (see Table 1) as the gross aggregate value per activity group (MXN$477,213 for primary activities; MXN$5,161,666 for industrial activities; MXN$10,003,740 for services) and total (MXN$15,642,620 value added). The second step was to define the accelerator $(\widehat{k})$, rather than as a diagonal matrix with capital-product sectoral margin ratios, as a constant for all sectors with a value of 7.[14] The third step, the coefficients of desired installed production capacity (β) are a vector calculated according to whether the desired installed production capacity is equal to consumption in the initial period (1.0; 1.3 and 1.25). It would usually be quite an arduous and complicated empirical work. However, for the purposes of this work, a random method was chosen, ensuring that the DS/IO simulates the system's behavior into the future, based on initial production conditions as shown in Table 1 (MXN$779,742 for primary activities; MXN$13,639,102 for industrial activities, and MXN$13,223,804 for services) and the fact that final demand levels are known.

The time series of final demand for the period of 2013-2019 were obtained as the GDP's percentage rate of change compared to the same quarter the previous year by type of sectoral activity (quarterly); a series deseasonalized at 2013 prices (INEGI). Table 3 presents this information.

Table 3. GDP percentage rate of change compared semesters year on year by type of sectoral activity

Year and semester	*Primary sector*	*Secondary sector*	*Tertiary sector*
2013.1	2.25	0.02	2.76
2	2.10	-0.11	2.05
2014.1	4.04	2.35	2.23
2	4.03	2.85	3.10
2015.1	3.37	1.01	4.26
2	1.77	1.51	4.27
2016.1	1.97	0.75	3.40
2	4.97	-0.29	3.89
2017.1	3.90	0.29	4.09
2	2.68	-0.39	2.52
2018.1	3.90	0.94	2.81
2	0.92	-0.05	2.96
2019.1	0.62	-1.85	0.87
2	5.37	-1.47	0.11

Source: INEGIhttps://www.inegi.org.mx/temas/pib/

Despite the "quality" of the parameters used, DS/IO can produce reliable projections of total GDP growth and by activity group. The real and projected values of these variables are compared in Table 4. Using the cumulative sum of average forecast errors (CFE) as a measure of forecast bias (projection minus real value) we see that the greatest underestimation occurs in the primary sector's GDP (17.9), while underestimation is lower in the secondary sector (1.4), and (7.1) in the tertiary sector and (4.0) in total GDP. Furthermore, we calculate the root mean square percentage error (RMSPE), which varies for the primary sector (18.8), dropping for the secondary sector (2.4), is (8.5) in the tertiary sector (8.5) and (5.0) for the total GDP. It works as a measure of the forecast's tendency to vary over time. As one can see from the data, only in the primary sector are there significant differences between the projected result and the real value.

Table 4. Real and projected GDP values by activity group and in total, 2013-2019 (semesters). Millions of pesos

Year and Semester	Primary Sector		Secondary Sector	
	Real	Projected	Real	Projected
2013.1	503402	303800	5174235	5161666
2	516369	456079	5201139	5253069
2014.1	523656	458360	5295659	5288181
2	537012	460652	5349469	5290292
2015.1	541224	462955	5349187	5292414
2	541757	465270	5430082	5294546
2016.1	551831	467596	5389069	5296689
2	568708	469934	5413984	5298843
2017.1	573322	472284	5404874	5301008
2	583981	474645	5392982	5303183
2018.1	583981	477018	5455794	5305369
2	595683	479403	5389986	5307567
2019.1	599411	481800	5354902	5309775
2	614618	532567	5343800	5312566
CPE	17.9		1.4	
RMSPE	18.8		2.4	
2013.1	9900169	7938636	15577806	13404102
2	10000334	10193366	15717842	15902514
2014.1	10120864	11474402	15940178	17220942
2	10310462	11531774	16196943	17282717
2015.1	10551870	11589433	16442282	17344801
2	10750754	11647380	16722592	17407196
2016.1	10909887	11705617	16850787	17469902
2	11168694	11764145	17151386	17532922
2017.1	11356340	11822966	17334536	17596257
2	11450454	11882081	17427417	17659909
2018.1	11675315	11999999	17715091	17782387
2	11788712	12001198	17774381	17788169
2019.1	11776597	12061204	17730909	17852780
2	11787068	12072120	17745486	17853985
CPE	7.1		4.0	
RMSPE	8.5		5.0	

Source: "Real" GDP values by sector activity group and GDP are available at https://www.inegi.org.mx/temas/pib/

To summarize, a refinement of the parameter's statistical data will improve the predictive capacity of the DS/IO, i.e. the quality of projections. However, the GDP growth forecast adjustment is satisfactory.

6. CONCLUSIONS

This work shows that while there are higher costs in opting for an unbalanced dynamic multisectoral model, the benefits can outweigh them. The advantages are mainly that the production and investment lag scheme better reflects the delays caused by the process of adjusting the goods market and that the investment projects' gestation period allows for surplus in production, varying degrees of installed productive capacity and maintaining the system's economic sustainability. The DS/IO offers a more complete model from the point of view of incorporating the actual processes of operation.

Second, it is possible to calculate dynamic multipliers using a DS/IO. In general, the magnitude of the dynamic multiplier is positively correlated with the ratio of intermediate goods to total production. In other words, the sectoral multiplier will be higher when the ratio of intermediate goods used in total production is greater. We noted that depending on the capital-product ratio for the sector, the convergence rate is different. In sectors characterized by a higher proportion of product capital, such as the secondary and tertiary, multiplier convergence rates will be lower.

Third, reliable GDP growth projections are possible in spite of the “quality” of the parameters used in the DS/IO. Real and projected values for GDP growth over time are "reasonable".

Fourth, DS/IO is a useful tool in exploring the possibilities for economic growth and methods to accelerate that process and make it more efficient.

Finally, it must be recognized that DS/IO also has its limitations. Among the most important are: *1)* not all investment can be explained by production levels; much of the investment consists of infrastructure works, military projects and stocks maintained for speculative purposes which are not directly linked to production; *2)* the lags framework in the investment or capital formation process is quite complicated and depends on technological, psychological, social, political, and institutional factors; and *3)* there is a plethora of ways to adjust excess production such as working additional shifts, expanding or building new facilities, and introducing technological changes.

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[1] The solution to this system is a constant game of cat and mouse. For the solution to be convergent, the cat (adjustment to production) must catch the mouse (demand surplus) (Johnson, 1986, p. 179).

[2] With cyclical periods characterized by drops in demand, net investment can be negative, but it is guaranteed to be equal to or greater than the depreciation, limited to non-negative gross investment and production values (Johnson, 1986, p. 180).

[3] Unfortunately, it is difficult to obtain GDP information by sectoral activity and for the whole period of time needed in order to validate the DS/IO at a subnational level. The information is also of inferior quality, to the point that the use of national information provides an advantage when it comes to statistical consistency and availability.

[4] Leontief (1953) suggested making investment endogenous in order to transform the static intersectoral model into a dynamic one. However, the endogenization of investment did not change the fact that the balance equation still described a long-term equilibrium. In other words, his proposed model describes the conditions in which the economy is in equilibrium but does not indicate how to achieve it (Johnson, 1986).

[5] This is a practical guide to become acquainted with and to manage the STELLA visual modeling software. It provides a referential framework and a GUI for quantitative and qualitative interaction and observation of a dynamic system's variables (Cervantes *et al.,* 2007).

[6] Desired inventories are considered to be equal to 1.0*consumption which means that an increase in consumption results in a production response of 2.0*consumption until inventories are first restored and then reach the new desired level. As such, the dynamic response is 2.0 times consumption minus inventory (Johnson *et al.,* 2008).

[7] The productive capacity limitation is imposed, making production equal to the minimum consumption requirements (including replenishing inventory) and each sector's production capacity (Johnson *et al.*, 2008).

[8] In STELLA, operation of the matrix multiplication, defined as matrix algebra, is not programmed directly. As the software runs by rows, matrices and vectors must be transposed before multiplication (Cervantes *et al.*, 2007)

[9] Rigorously estimating DS/IO parameters would imply an empirical work more complex and arduous in nature. Nevertheless, for the purposes of the current work we opted for a random method, ensuring that the DS/IO "reasonably" simulates the system's behavior.

[10] The value used was a constant equal to 7 while Johnson (1986) uses a value equal to 5.

[11] The value used is a constant equal to (0.1) for each sector.

[12] In figure 1 one can see that the auxiliary variables as well as converters, are represented by a circle. Meanwhile, figure 2 shows that when they are defined in terms of matrices they are represented by a shaded circle.

[13] As was previously established, using national information guarantees the information's statistical consistency.

[14] The accelerator reflects the structure of projected lag in the investment gestation period. The inclusion of a complicated lag structure makes the DS/IO difficult to understand and manage, and can lead to problems of instability and strange behavior (Kozikowski, 1988, p. 192).